NSF-ITP-01-16

Comments on supergravity dual of pure $\mathcal{N}=1$ Super Yang Mills theory with unbroken chiral symmetry

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Abstract

Maldacena and Nunez [hep-th/0008001] identified a gravity solution describing pure $\mathcal{N}=1$ Yang-Mills (YM) in the IR. Their (smooth) supergravity solution exhibits confinement and the $U(1)_R$ chiral symmetry breaking of the dual YM theory, while the singular solution corresponds to the gauge theory phase with unbroken $U(1)_R$ chiral symmetry. In this paper we discuss supersymmetric type IIB compactifications on resolved conifolds with torsion. We rederive singular background of [hep-th/0008001] directly from the supersymmetry conditions. This solution is the relevant starting point to study non-BPS backgrounds dual to the high temperature phase of pure YM. We construct the simplest black hole solution in this background. We argue that it has a regular Schwarzschild horizon and provides a resolution of the IR singularity of the chirally symmetric extremal solution as suggested in [hep-th/0011146].

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1 Introduction

The AdS/CFT duality of Maldacena [1] is a very useful tool in study of nonperturbative dynamics of four dimensional gauge theories. The main idea of the approach is to use a dual gravitational description of the gauge theory living on a large stack of N coincident D3-branes in string theory. When D3 branes are placed in a smooth type IIB background, the string theory in the near horizon geometry of the stack is dual to $\mathcal{N} = 4$ supersymmetric YM theory [1, 2, 3].

Several approaches are used to construct gravitational backgrounds dual to gauge theories with reduced supersymmetry (and thus more interesting IR dynamics) [4]. In particular, $\mathcal{N} = 1$ gauge theory can be obtained by mass deformation of parent $\mathcal{N} = 4$ gauge theory, placing large number of D3 branes on appropriate conical singularity, or, as suggested by Maldacena and Nunez [5], in the IR of a little string theory realized by wrapping NS 5 branes of type IIB string theory on a 2-cycle, and appropriately twisting the normal bundle to preserve 1/4 of original supersymmetries. Typically, in gravitational dual of nonconformal gauge theories with reduced supersymmetry one encounters naked singularities in the IR region. Over the last year we learned that these naked singularities potentially signal to an interesting physical phenomenon in the IR dynamics of gauge theories. The nontrivial gauge theory IR physics often has a gravity (or string theory) dual that resolves the naked singularity. Alternatively, understanding the resolution of naked singularities in gravitational backgrounds [1] could teach us about nonperturbative effects in the gauge theory.

For example, from the chiral symmetry breaking in the IR of the gauge theory on the world volume of regular and fractional branes on the conifold, Klebanov and Strassler [6] argued that the naked singularity of the Klebanov-Tseytlin (KT) geometry [7] should be resolved via the deformation of the conifold. "The flow of information" in the opposite direction, i.e. from the string theory to the field theory, was proposed in [8]. It was

[1] Not all naked singularities are physical and can be resolved.

suggested there that naked singularity in the KT geometry could be alternatively resolved by placing sufficiently large black hole in the background, so that its horizon would cloak the naked singularity. "Sufficiently large" means that the Hawking temperature of the black hole horizon should be larger than the critical temperature of the chiral symmetry breaking phase transition. Here, the hope is that, by studying the black hole of the critical radius one would learn about the gauge theory phase transition. The black hole solution proposed in [8] fails to realize this scenario. As shown in [9], the horizon of the non-extremal solution presented in [8] does not cloak the singularity, but rather coincides with it. This type of singular horizon is deemed unacceptable in studies of black hole metrics. In [9] a system of second order equations is derived whose solutions may describe non-extremal generalizations of the KT background with regular horizons. Recently constructed smooth solutions to this system [10] in a perturbation theory valid for large Hawking temperature of the horizon show that this is indeed possible. These solutions appear to support the suggestion of [8] that a regular horizon of the non-extremal generalization of the KT geometry appears only at some finite Hawking temperature.

Gravitational backgrounds which regular Schwarzschild horizon exists only above some critical non-extremality are unusual and, to our knowledge, new from the supergravity point of view. Analysis of [8, 9, 10] suggests that they should nonetheless be quite generic for backgrounds dual (in Maldacena sense) to gauge theories which undergo finite temperature symmetry breaking phase transition. It is thus of interest to look for additional examples of this phenomenon. An obvious choice is the supergravity solution constructed by Chamseddine and Volkov in [11, 12] and interpreted by Maldacena and Nunez [5] as a gravity dual of the pure $\mathcal{N} = 1$ supersymmetric Yang Mills theory in the IR. In what follows we refer to this supergravity background as CV-MN. The smooth solution of [11, 12, 5] has the same IR behavior as that of the cascading gauge theory of [6], thus corresponding to the phase of the gauge theory with broken chiral symmetry at zero temperature. The singular CV-MN solution, like the KT geometry, describes the phase of the gauge

theory with the unbroken symmetry. The crucial difference between the two models is that KT-KS model has an effective four dimensional description in the UV as well [13], while the CV-MN model is regularized in the UV by the little string theory. As a first step towards understanding the thermodynamics and the phase transition in the CV-MN model we construct a non-BPS generalization of the chirally symmetric (singular) CV-MN background. We argue that presented solutions have a regular horizon that exists above some critical value of non-extremality, in agreement with the dual gauge theory where the chiral symmetry is restored at finite temperature.

As a separate issue, we will also extend a no-go theorem for supersymmetric compactifications with torsion [14, 17]. The work of [17] studied compactifications of type IIB supergravity of the form given by [14] to 4 Minkowski dimensions. They showed that for 6 compact internal dimensions, the only solutions with globally defined dilaton are Calabi-Yau three-folds with vanishing torsion. This no-go theorem can be avoided on noncompact internal manifolds, as the CV-MN solution shows. Our results extend the no-go theorem of [17] to noncompact internal manifolds with the complex structure of the resolved conifold, in that all BPS solutions will have a naked singularity.

This paper is organized as follows. In the next section following the work [14], we review the general construction of supersymmetric vacua of type IIB supergravity with torsion. In section 3 we study supersymmetric compactifications of type IIB supergravity on resolved conifolds (which is relevant to the unbroken chiral symmetry phase of the CV-MN model). We show that all SUSY preserving vacua have naked singularity. In section 4 we rederive a simple singular solution of [11, 12, 5]. In section 5 we discuss non-extremal generalizations of the background. We conclude with discussion in section 6.

2 Supersymmetric type IIB compactifications with torsion

Conditions for spacetime supersymmetry of the heterotic superstring on manifolds with torsion were found in [14]. In this section we consider corresponding conditions for type IIB compactifications to four dimensions on manifolds with torsion. As we set all the R-R fields to be zero, our analysis essentially repeat those of [14]. Similar to the heterotic compactifications, we find that supersymmetric type IIB vacua with torsion have warped four dimensional space-time and nontrivial dilaton.

Type IIB equations of motion and supersymmetry variations have been found in [15] which notation we follow here[2]. In particular, we use mostly negative signature for the metric. The massless bosonic fields of the type IIB superstring theory consist of the complex dilaton field B that parameterizes the $SL\left(2,\mathbf{R}\right)/U\left(1\right)$ coset space, the metric tensor g_{MN} and the antisymmetric complex 2-tensor $A_{(2)}$, and the four-form field $A_{(4)}$ with self-dual five-form field strength. Their fermionic superpartners are a complex Weyl gravitino ψ_M ($\hat{\gamma}^{11}\psi_M = -\psi_M$) and a complex Weyl dilatino λ ($\hat{\gamma}^{11}\lambda = \lambda$). The theory has $\mathcal{N}$=2 supersymmetry generated by two supercharges of the same chirality.

We would like to find bosonic backgrounds that preserve some supersymmetry. This will be the case provided the supersymmetry variation of the fermionic fields is zero

$$\begin{aligned}
\delta\lambda &= iP_M\hat{\gamma}^M\epsilon^* - \frac{i}{24}G_{MNP}\hat{\gamma}^{MNP}\epsilon\,,\\
\delta\psi_M &= \hat{D}_M\epsilon + \frac{i}{480}F_{P_1\cdots P_5}\hat{\gamma}^{P_1\cdots P_5}\hat{\gamma}_M\epsilon + \frac{1}{96}(\hat{\gamma}_M{}^{NPQ}G_{NPQ}\\
&\qquad -9\hat{\gamma}^{NP}G_{MNP})\epsilon^*\,,
\end{aligned} \tag{2.1}$$

where

$$F_{(5)} = dA_{(4)} - \frac{1}{8}\mathrm{Im}A_{(2)}\wedge F^*_{(3)}\,, \qquad F_{(3)} = dA_{(2)}\,,$$

[2]See Appendix for our detailed notations and conventions.

$$
\begin{aligned}
G_{(3)} &= \frac{F_{(3)} - BF^*_{(3)}}{\sqrt{1-|B|^2}}\,, \qquad P_M = \frac{\partial_M B}{1-|B|^2}\,, \\
\hat\gamma^{11}\epsilon &= -\epsilon\,.
\end{aligned} \tag{2.2}
$$

The covariant derivative $\hat D_M$ contains $U(1)$ connection $Q_M = \mathrm{Im}(B\partial_M B^*)/(1-|B|^2)$

$$
\hat D_M\epsilon = \left(\hat\nabla_M - \frac{1}{2} i Q_M\right)\epsilon = \left(\partial_M - \frac{1}{4}\hat\omega_{MNP}\hat\gamma^{NP} - \frac{1}{2} i Q_M\right)\epsilon\,. \tag{2.3}
$$

The spin connection is given by

$$
\hat\omega_{MNP} = \hat e^r_{M[}\partial_N \hat e_{P]r} + \hat e^r_{P[}\partial_N \hat e_{M]r} + \hat e^r_{N[}\partial_M \hat e_{P]r}\,. \tag{2.4}
$$

The above supergravity potentials and the dilaton field differ from those conventionally used in D-brane physics. In the latter case we have the dilaton ϕ, $B_{(2)}$ two-form from the NS-NS sector, and $C_{(n)}$ forms from the R-R sector with $n = 0, 2, 4, 6, 8$. The dictionary between the two descriptions was presented in [16], which we adopt here. In particular we have

$$
\begin{aligned}
C_{(0)} + i e^{-\phi} &= i\frac{1+B}{1-B}\,, \\
A_{(2)} &= C_{(2)} + i B_{(2)}\,.
\end{aligned} \tag{2.5}
$$

It is easy to see that in type IIB equations of motion we can consistently set all R-R potentials to zero. Thus we have

$$
\begin{aligned}
&G_{MNP} = i H_{MNP} e^{-\phi/2}\,, \qquad P_M = -\frac{1}{2}\partial_M\phi\,, \\
&F_{P_1\cdots P_5} = 0\,, \qquad Q_M = 0\,,
\end{aligned} \tag{2.6}
$$

where $H = dB_{(2)}$. We assume geometry to be a direct product of two spaces $M_4 \times K$

$$
d\hat s^2_{10} = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu - e^{2B(y)} g_{mn} dy^m dy^n\,, \tag{2.7}
$$

with warp factors depending on the coordinates of the six dimensional factor only. The warp factor $B(y)$ is not to be confused with the complex dilaton

parameterizing $SL(2,\mathbf{R})/U(1)$ coset space; we will not use the latter in the following. Furthermore, we take both the dilaton and the NS-NS two form to depend only on the coordinates y^m of K. Introducing

$$\epsilon = e^{i\pi/4}\eta\,, \qquad \eta^* = \eta\,, \tag{2.8}$$

and manipulating with gamma matrices, supersymmetry variations (2.1) become

$$\begin{aligned}
\delta\lambda &= \left[-\frac{1}{2}\hat{\partial\!\!\!/}\phi\!\!\!/ + \frac{1}{4}e^{-\phi/2}\hat{H\!\!\!/}\right]\eta\,,\\
\delta\psi_M &= \hat{\nabla}_M\eta - \frac{e^{-\phi/2}}{16}\left(2\hat{H\!\!\!/}\hat{\gamma}_M + \hat{\gamma}_M\hat{H\!\!\!/}\right)\eta\,.
\end{aligned} \tag{2.9}$$

Vanishing of the dilatino and gravitino variations (2.9) is identical to the supersymmetry preserving conditions of the heterotic compactification with torsion discussed in [14], provided we set gauge fields to zero. So we can simply adopt the latter results. First of all, from the gravitino variation in four dimensions we find

$$A = -\frac{1}{4}\phi\,, \tag{2.10}$$

that is, the M_4 factor of the background geometry is flat Minkowski space in string frame. The remaining SUSY preserving conditions is convenient to formulate in string frame, that is choosing

$$B = -\frac{1}{4}\phi\,. \tag{2.11}$$

Consider six dimensional manifold $\tilde{K}$, with the metric

$$ds^6_{\tilde{K}} = g_{mn}dy^m dy^n\,. \tag{2.12}$$

Unbroken supersymmetry in four dimensions implies $\tilde{K}$ to be hermitean, endowed with a holomorphic (3,0) form ω [14]:

$$\bar{\partial}\omega = 0\,. \tag{2.13}$$

Introducing global complex coordinates z^i and $\bar{z^i} \equiv z^{\bar{i}}$ on $\tilde{K}$ and denoting global (anti-)holomorphic indices as $(\bar{a}, \bar{b}, \bar{c}, \cdots)$ $a, b, c, \cdots$, the fundamental (1,1) form on $\tilde{K}$

$$J = i g_{a\bar{b}} dz^a \wedge dz^{\bar{b}} \tag{2.14}$$

relates to ω as

$$d^+ J + i \left(\bar{\partial} - \partial \right) \ln ||\omega|| = 0 \,, \tag{2.15}$$

where the norm of ω is given by

$$||\omega||^2 = \omega_{a_1 a_2 a_3} \bar{\omega}_{\bar{b}_1 \bar{b}_2 \bar{b}_3} g^{a_1 \bar{b}_1} g^{a_2 \bar{b}_2} g^{a_3 \bar{b}_3} \,. \tag{2.16}$$

Finally, the torsion is

$$H = i \left(\bar{\partial} - \partial \right) J \,, \tag{2.17}$$

and the dilaton is given by

$$\phi = \phi_0 - \frac{1}{2} \ln ||\omega|| \,, \tag{2.18}$$

where we explicitly included constant ϕ_0.

3 Supersymmetric compactifications on resolved conifolds with torsion

[3] Supersymmetric compactifications on Hermitian manifolds with torsion are very restrictive. In fact, in [17] it was shown that there are no supersymmetric compactifications of this type on compact Hermitian manifolds M^6 with non-vanishing torsion and a globally defined dilaton. This no-go theorem could be avoided with non-compact M^6 [5]. It is thus of interest to study supersymmetry on other non-compact Hermitian manifolds with torsion.

In this section we discuss supersymmetric compactifications of type IIB supergravity on resolved conifolds with torsion. We explicitly show that

[3]Results reported in this section we obtained in collaboration with Joe Polchinski.

with the nonvanishing torsion, the resolved conifold geometry always has a naked singularity, also the dilaton can not be defined globally. As shown in [5] supersymmetric backgrounds in this class are dual to the phase with unbroken chiral symmetry of pure $\mathcal{N}=1$ four dimensional Yang Mills theory at zero temperature. Since the chiral symmetry of the YM theory is broken in the IR, the appearance of a naked singularity is not surprising.

The general supersymmetry conditions for type IIB compactification with torsion were reviewed in the previous section. The string frame metric is given by

$$ds^2_{str} = \eta_{\mu\nu} dx^\mu dx^\nu - ds^2_6 \,, \tag{3.19}$$

where the hermitian metric ds_6^2 on the six dimensional manifold $\tilde{K}$ (resolved conifold in our case) satisfies (2.15) once the holomorphic $(3,0)$ form ω is specified. The SUSY preserving torsion is then given by (2.17), and the dilaton is determined from (2.18). After recalling some useful facts from the conifold geometry [21], we solve (2.15). The metric is further constraint by the Bianchi identity on the torsion

$$dH = \rho_5 \,, \tag{3.20}$$

where ρ_5 is the properly normalized density of the NS5 branes. In this paper we assume $\rho_5 = 0$, except for possible delta-function sources.

A singular conifold can be described by a quadric in $\mathbb{C}_4$

$$XY - UV = 0 \,. \tag{3.21}$$

A small resolution of the cone is obtained by replacing the equation (3.21) by the pair of equations

$$\tilde{K}: \qquad W \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = 0 \,, \qquad \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} \in \mathbb{P}_1 \,, \tag{3.22}$$

where the matrix

$$W = \begin{pmatrix} X & U \\ V & Y \end{pmatrix} \,, \tag{3.23}$$

has rank 1, except when all of X, Y, U, V vanish where it has rank 0. Away from the apex of the cone (3.22) determines a unique point on $\mathbb{P}_1$. At the apex

of the cone (3.22) defines the entire $\mathbb{P}_1$. Let $H_+$and H_- be two coordinate patches covering $\mathbb{P}_1$ with local coordinates $\lambda \equiv \frac{\lambda_2}{\lambda_1}$, $\lambda_1 \neq 0$ and $\mu \equiv \frac{\lambda_1}{\lambda_2}$, $\lambda_2 \neq 0$ respectively. On H_+, (3.22) implies

$$W = \begin{pmatrix} -U\lambda & U \\ -Y\lambda & Y \end{pmatrix}, \tag{3.24}$$

so we can choose (U, Y, λ) as our complex coordinates. Similarly, on H_- we have

$$W = \begin{pmatrix} X & -X\mu \\ V & -V\mu \end{pmatrix}, \tag{3.25}$$

so we can choose as complex coordinates (V, X, μ). On $H_+ \cap H_-$ the transition function is given by

$$(V, X, \mu) = (-Y\lambda, -U\lambda, 1/\lambda)\,. \tag{3.26}$$

In above coordinates the holomorphic $(3, 0)$ form takes a simple form

$$\omega = dU \wedge dY \wedge d\lambda = dV \wedge dX \wedge d\mu\,. \tag{3.27}$$

Resolved conifold has global $SU(2) \times SU(2)$ symmetry with the following action

$$W \to LWR^+\,,$$
$$\begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} \to R \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix}, \tag{3.28}$$

where L, R are independent $SU(2)$ matrices. The three form ω of (3.27) is invariant under the symmetries of the resolved conifold, and up to a c-number factor is unique[4].

The most general $SU(2) \times SU(2)$ invariant Hermitian metric on $\tilde{K}$ takes the form

$$ds_6^2 = f_1 \,\mathrm{tr}\left(dW^+ dW\right) + f_2\, |\mathrm{tr}\left(W^+ dW\right)|^2 + f_3\, \frac{|d\lambda|^2}{(1 + |\lambda|^2)^2}\,, \tag{3.29}$$

[4]Any other holomorphic three form $\tilde{\omega}$ would differ from (3.27) by a holomorphic $SU(2)\times SU(2)$ invariant function $f(W)$: $\tilde{\omega} = f(W)\omega$, where W is given by (3.24), (3.25) on H_+, H_- correspondingly. With the transformation law (3.28), f can depend only on $\det[W]$: $f(W) \equiv \tilde{f}(\det[W]) = \tilde{f}(0)$, which is constant.

where $f_i = f_i(x)$ are scalar functions of

$$x \equiv \mathrm{tr}\left(W^+W\right) . \tag{3.30}$$

Consider first condition coming from the gravitino variation (2.15). The norm of the holomorphic three form (3.27) in (3.29) evaluates to

$$||\omega|| = \sqrt{6}\, \det{}^{-1/2}(g_{a\bar{b}}) = \sqrt{6}\, \Big[f_1(f_1+f_2x)(f_1x+f_3)\Big]^{-1/2} . \tag{3.31}$$

As a shorthand notation we define

$$g \equiv \det(g_{a\bar{b}}) = f_1(f_1+f_2x)(f_1x+f_3) . \tag{3.32}$$

Now,

$$i\left(\bar{\partial}-\partial\right)\ln||\omega|| = -\frac{i}{2}\,[\ln g]'\left[\mathrm{tr}\left(dW^+W\right) - \mathrm{tr}\left(W^+dW\right)\right] , \tag{3.33}$$

where the prime denotes derivative with respect to x. After some straightforward, though rather tedious algebra we find

$$d^+J = i\,\frac{2f_1f_1'x + f_3f_1' + f_1f_3' - f_2f_3 - 2f_1f_2x}{f_1(f_1x+f_3)}\,\left[\mathrm{tr}\left(dW^+W\right) - \mathrm{tr}\left(W^+dW\right)\right] . \tag{3.34}$$

With (3.33) and (3.34), eq. (2.15) gives

$$\Big[f_1(f_1+f_2x)(f_1x+f_3)\Big]' + 2(f_1+xf_2)\Big[(f_2-f_1')(f_3+2f_1x) - f_1f_3'\Big] = 0 . \tag{3.35}$$

Once the metric (3.29) is specified, we can determine torsion following (2.17). We will not give the complete expression here, just mention that the Bianchi identity on H (in the absence of NS5 branes) results in the following constraints

$$\begin{aligned} \Big[(f_1'-f_2)x^2\Big]' &= 0 , \\ f_3' &= 2x(f_2-f_1') . \end{aligned} \tag{3.36}$$

Finally, the dilaton is determined by (2.18)

$$\phi = \text{const} + \frac{1}{4}\ln\Big[f_1(f_1+f_2x)(f_1x+f_3)\Big]\,. \tag{3.37}$$

Eqs. (3.35) and (3.36) represent complete set of constraints that determine supersymmetric compactifications on type IIB string theory on resolved conifolds with torsion.

To write down the metric (3.29) explicitly we will parameterize W in terms of two sets of Euler angles[5]

$$\begin{aligned} U &= re^{i(\psi+\phi_1+\phi_2)/2}\cos\frac{\theta_1}{2}\cos\frac{\theta_2}{2}\,,\\ Y &= re^{i(\psi-\phi_1+\phi_2)/2}\sin\frac{\theta_1}{2}\cos\frac{\theta_2}{2}\,,\\ \lambda &= e^{-i\phi_2}\tan\frac{\theta_2}{2}\,. \end{aligned} \tag{3.38}$$

The metric on $\tilde{K}$ is then given by

$$\begin{aligned} ds_6^2 = (dr)^2\,(f_1+f_2x) + \frac{f_1x}{4}\left(d\theta_1^2+\sin^2\theta_1\ d\phi_1^2\right) + \frac{f_1x+f_3}{4}\left(d\theta_2^2+\sin^2\theta_2\ d\phi_2^2\right)\\ +\frac{f_1x+f_2x^2}{4}\left(d\psi+\sum_{i=1}^{2}\ \cos\theta_i\ d\phi_i\right)^2\,, \qquad x=r^2\,. \end{aligned} \tag{3.39}$$

From the Bianchi constraints (3.36), away from the 5-brane sources, we find

$$[f_3'x]' = 0\,, \tag{3.40}$$

with the most general solution

$$f_3 = c_1\ln x + c_2\,, \tag{3.41}$$

for some constants c_i. Furthermore,

$$f_2 = f_1' + \frac{f_3'}{2x}\,. \tag{3.42}$$

[5] This parametrization first appeared in [18].

Given (3.41) and (3.42), (3.35) gives an ordinary (nonlinear) differential equation on f_1. Some general conclusions concerning supergravity backgrounds discussed here could be reached without explicitly solving the resulting equation. Most importantly, with (3.41) we immediately see from (3.39) that unless $c_1 = 0$ the background geometry always has a naked singularity: radius squared of one of the two S^2s (parameterized by (θ_i, ϕ_i) for $i = 1$ or $i = 2$) will necessarily become negative for $r > r_s$ (or $r < r_s$ depending on the sign of c_1) with some r_s. From (3.37), we see that in the same region the dilaton would become complex valued. When $c_1 = 0$ and c_2 being arbitrary, eq. (3.42) requires $\tilde{K}$ to be Kähler; we find in this case torsion to vanish identically. This is the main result of the section: we showed that there are no nonsingular supersymmetric backgrounds of type IIB supergravity on resolved conifolds with nontrivial torsion.

We can also make a few comments about the nature of this naked singularity. First of all, following the definition of [19], we can see that this is a repulson singularity. That is, a massive particle coupled to the Einstein metric follows a radial trajectory given by

$$\tau = \int dr |g_{rr}|^{1/2} g_{tt}^{1/2} \left[E^2 - g_{tt} \right]^{-1/2} , \tag{3.43}$$

where E is the energy per unit mass. Because $g_{tt} = e^{-\phi/2}$ blows up at the singularity (recall (3.37)), the particle will always bounce away from the singularity in finite proper time. This also implies that our solutions violate the criterion of [20], because g_{tt} in the Einstein frame is unbounded at the singularity. This means they cannot accurately describe the IR dynamics of a dual gauge theory. This is not to say that our singularities cannot be resolved (indeed the CV-MN solution is the resolution) but rather that there is no chirally symmetric phase to the dual gauge theory at extremality.

4 Supergravity dual of YM theory with unbroken chiral symmetry

In [5] Maldacena and Nunez described gravitational solutions corresponding to a large number of NS fivebranes wrapping a two sphere. They argued that these solutions describe pure $\mathcal{N}=1$ super YM in the IR. More specifically, they described two solutions: one having a smooth geometry and broken $U(1)_R$ chiral symmetry of the dual gauge theory in the IR[6], and a solution with a naked singularity in the IR, dual to the gauge theory phase with the unbroken chiral symmetry. The latter could be understood as a supersymmetric vacuum of type IIB string theory on the resolved conifold with torsion. Using the results of the previous section, we show here that this is indeed the case.

A simple solution of (3.35), (3.36) is

$$\begin{aligned} f_3 &= -2a^2 \ln \frac{x}{r_*^2}\,, \\ f_2 &= -\frac{2a^2}{x^2} \ln \frac{x}{r_*^2}\,, \\ f_1 &= \frac{a^2}{x} \left(1 + 2\ln \frac{x}{r_*^2}\right)\,, \end{aligned} \tag{4.44}$$

where a, r_* are constants. The ten-dimensional metric in the string frame is given by

$$\begin{aligned} ds^2_{str} = \eta_{\mu\nu} dx^\mu dx^\nu - a^2 \Bigg[\frac{(dr)^2}{r^2} + \frac{1 + 2\ln \frac{r^2}{r_*^2}}{4} \left(d\theta_1^2 + \sin^2\theta_1 \; d\phi_1^2\right) \\ + \frac{1}{4}\left(d\theta_2^2 + \sin^2\theta_2 \; d\phi_2^2\right) + \frac{1}{4}\left(d\psi + \sum_{i=1}^{2} \cos\theta_i \; d\phi_i\right)^2 \Bigg]\,. \end{aligned} \tag{4.45}$$

The torsion is

$$H = \frac{a^2}{4}\left[\left(d\psi + \sum_{i=1}^{2} \cos\theta_i \; d\phi_i\right) \wedge \left(\sin\theta_1 \; d\theta_1 \wedge d\phi_1 - \sin\theta_2 \; d\theta_2 \wedge d\phi_2\right)\right]\,, \tag{4.46}$$

[6] The complex geometry and the supersymmetry of the smooth solution has been discussed in details in [23].

and the dilaton

$$e^{\phi} = \text{const} \left(\frac{1 + 2\ln \frac{r^2}{r_*^2}}{r^4/a^4} \right)^{1/4} . \tag{4.47}$$

Note that the naked singularity is at r_s

$$r_s = r_* e^{-1/4} . \tag{4.48}$$

We would like to match (4.45)-(4.47) with the UV behavior of the smooth CV-MN solution. The smooth ten dimensional solution in [5] is given by

$$\begin{aligned}
ds^2_{str} &= \eta_{\mu\nu} dx^\mu dx^\nu - N \Big[d\rho^2 + e^{2g(\rho)} \left(d\theta^2 + \sin^2\theta \; d\varphi^2 \right) \\
&\quad + \frac{1}{4} \sum_a (w^a - A^a)^2 \Big] , \\
e^{2\phi} &= \text{const} \; \frac{2e^{g(\rho)}}{\sinh 2\rho} ,
\end{aligned}$$

$$H = \frac{N}{4} \left[-(w^1 - A^1) \wedge (w^2 - A^2) \wedge (w^3 - A^3) + \sum_a F^a \wedge (w^a - A^a) \right] , \tag{4.49}$$

where

$$\begin{aligned}
A^1 &= a(\rho) , \qquad A^2 = a(\rho) \sin\theta , \qquad A^3 = \cos\theta , \\
a(\rho) &= \frac{2\rho}{\sinh 2\rho} , \\
e^{2g} &= \rho \coth 2\rho - \frac{\rho^2}{\sinh^2 2\rho} - \frac{1}{4} , \\
w^1 + i w^2 &= e^{-i\psi} \left(d\tilde{\theta} + i \sin\tilde{\theta} d\phi \right) , \qquad w^3 = d\psi + \cos\tilde{\theta} d\phi .
\end{aligned} \tag{4.50}$$

In the limit $\rho \to \infty$ we find that both backgrounds agree, provided we identify

$$\begin{aligned}
\rho &= \ln r , \qquad \text{as } r \to \infty , \\
N &= a^2 .
\end{aligned} \tag{4.51}$$

5 Towards finite temperature resolution of the IR singularity

In the previous section we showed that the supergravity solution described by Maldacena and Nunez [5] with unbroken $U(1)_R$ symmetry is in the class of type IIB compactifications on resolved conifold with nonvanishing torsion. All such solutions have a naked singularity in the IR which is the reflection of the fact that the chiral symmetry in the dual Yang Mills *must* be broken at low energies. In fact, the smooth CV-MN solution has this $U(1)_R$ symmetry broken to a Z_2, as predicted by the dual gauge theory.

Rather similarly, the naked singularity of the KT geometry is resolved in [6]. A different mechanism for resolving this singularity was proposed in [8]. As we expect that the chiral symmetry of the gauge theory is restored at a finite temperature T_c, we expect that there should exist a non-extremal generalization of the KT geometry with the regular horizon cloaking the singularity. Such regular Schwarzschild horizon should appear only for some finite Hawking temperature. This is a rather unusual phenomenon from the supergravity point of view. The non-BPS generalization presented in [8] does not realize this proposal. As shown in [9], the horizon of the solution discussed there is singular for arbitrary non-extremality parameter. The horizon singularity can be traced back [9] to the too restrictive requirement of the self-duality of the three form fluxes off the extremality. On a more technical level, this corresponds to the fact that the $U(1)$ fiber of compact $T^{1,1}$ in the KT geometry was not "squashed" relative to the 2-spheres of $T^{1,1}$. Such squashing does not violate $U(1)_R$ chiral symmetry and is necessary for a non-BPS solution to have a nonsingular horizon. This has been shown in [10]. There, the daunting task of solving a coupled system of the second order differential equations describing regular non-extremal generalization of the KT geometry was approached with a beautiful physical insight: in the KT-KS model the number of fractional D3 branes is fixed, while the number or regular ones changes logarithmically with the energy scale (radial coordinate); thus, if at the horizon of the non-BPS KT geometry the number of regular

D3 branes is still large, one could imagine developing a perturbation theory around standard black D3 branes [22], with the small parameter being the ratio of fractional and the regular D3 branes. Computation to the first order in this perturbation theory demonstrated the chiral symmetry restoration in the gravitational dual of the cascading gauge theory at high temperature, with horizon "cloaking" the naked singularity of the extremal KT solution [10].

In this section we construct nonsingular, non-BPS generalizations of the geometry (4.45)-(4.47). In the case of $T^{1,1}$ of the KT geometry, squashing the $U(1)$ fiber off the extremality induced a source for the dilaton [9, 10]. That is, the constant dilaton at $T = 0$, should run for $T \geq T_c$. In our case, the dilaton is nontrivial even at the extremality. In constructing the appropriate non-BPS solution we assume that the $U(1)$ fiber (parameterized by ψ in (4.45)) is not squashed relative to the (θ_2, ϕ_2) sphere, as it is at the extremality. This restriction substantially simplifies type IIB equations of motion, but unlike a somewhat similar ansatz in [8], leads to geometries with regular horizons.

In what follows we discuss the following non-extremal generalization of (4.45)-(4.47):

$$
\begin{aligned}
ds_E^2 &= c_1(r)^2\left[\triangle_1^2 dt^2 - d\bar{x}^2\right] - c_1(r)^2 a^2\left[\frac{dr^2}{\triangle_2^2 r^2} + \frac{h(r)}{4}\left(d\theta_1^2 + \sin^2\theta_1\ d\phi_1^2\right)\right.\\
&\qquad \left. + \frac{1}{4}\left(d\theta_2^2 + \sin^2\theta_2\ d\phi_2^2\right) + \frac{1}{4}\left(d\psi + \sum_{i=1}^{2}\ \cos\theta_i\ d\phi_i\right)^2\ \right],\\
H &= \frac{a^2}{4}\left[\left(d\psi + \sum_{i=1}^{2}\ \cos\theta_i\ d\phi_i\right)\wedge\left(\sin\theta_1\ d\theta_1\wedge d\phi_1 - \sin\theta_2\ d\theta_2\wedge d\phi_2\right)\right],\\
\phi(r) &= \phi(r)\,,
\end{aligned}
\tag{5.52}
$$

where the metric is given in Einstein frame, ϕ in the last equation denotes the dilaton. Note that we used the same torsion as in the extremal case (4.46).

Checking type IIB equations of motion is tedious, so we only outline the

steps. We perform the computations in the orthonormal frame with

$$
\begin{aligned}
& e^1 = c_1 \triangle_1 dt\,, \qquad e^{2\cdots4} = c_1 dx^{1\cdots3}\,, \qquad e^5 = c_1 a\,\frac{dr}{\triangle_2 r}\,, \\
& e^6 = \frac{c_1 a}{2}\,\left(d\psi + \sum_{i=1}^{2}\,\cos\theta_i\;d\phi_i\right)\,, \qquad e^7 = \frac{c_1 a h^{1/2}}{2}\;d\theta_1\,, \\
& e^8 = \frac{c_1 a h^{1/2}}{2}\;\sin\theta_1\;d\phi_1\,, \qquad e^9 = \frac{c_1 a}{2}\;d\theta_2\,, \\
& e^{10} = \frac{c_1 a}{2}\;\sin\theta_2\;d\phi_2\,.
\end{aligned}
\tag{5.53}
$$

With ansatz (5.52), the 3-form Maxwell equations and its Bianchi identity are satisfied automatically. The energy-momentum tensor of the three form satisfies

$$
T_{11}^{(3)} + T_{22}^{(3)} = 0\,,
\tag{5.54}
$$

which requires the sum of the corresponding components of the Ricci tensor to vanish. This gives

$$
\triangle_1' \triangle_2 = \frac{A}{c_1(r)^8 h(r) r}\,,
\tag{5.55}
$$

where (a constant) A is the non-extremality parameter. Turns out, all the other Einstein equations, and the dilaton equation, could be reduced to the following second order differential equations

$$
\begin{aligned}
0 &= A^2\Big[\ln F(y)\Big]'' - 4F^2(y)\left(h^2(y)+1\right)\,, \\
0 &= A^2\Big[\ln h(y)\Big]'' - 8F^2(y)\left(h(y)-1\right)\,, \\
0 &= A^2\Big(\Big[F^2(y)\Big]'\Big[h^2(y)\Big]' + F^2(y)\left(h(y)'\right)^2 + 2h^2(y)\left(F(y)'\right)^2 \\
&\qquad -2F^2(y)h^2(y)\Big) - 8F^4(y)h^2(y)\left(h^2(y)+2h(y)-1\right)\,,
\end{aligned}
\tag{5.56}
$$

where the derivatives are with respect to[7]

$$
y \equiv \ln \triangle_1(r)\,,
\tag{5.57}
$$

[7]Note that the "good" coordinate of [9, 10] is also proportional to $\ln \triangle_1$.

also

$$F(y(r)) = c_1^8(r)\triangle_1(r)\,, \qquad h(r) = h(y(r))\,,$$
$$e^{-\phi/2} = c_1^2(r)\,. \tag{5.58}$$

Note that dilaton is related to the warp factor of three space dimensions $\bar{x}$ as in the extremal case.

The system of differential equations (5.56) is overdetermined. It is straightforward to check that it is actually compatible. Further assuming $F(y) = F(h(y))$ one can obtain from (5.56) a second order nonlinear differential equation for $F(h)$

$$\begin{aligned} &F''Fh^2\left[A^2+4F^2\left(h^2+2h-1\right)\right]+F'h\Big[-F\left(4F^2\left(h^2-3h+4\right)-A^2\right)\\ &-F'h\left(8F^2\left(h^2-h+2\right)+A^2\right)+8F'^2Fh^2(h-1)\Big]\\ &-2F^4(h^2+1) \;=\; 0\,, \end{aligned} \tag{5.59}$$

where the derivatives are with respect to h. Above equation is solved with

$$F(h) = C\ h^{-1/2}e^{h/2}\,, \tag{5.60}$$

for zero non-extremality parameter $A = 0$, thus reproducing (4.45)-(4.47) directly from the type IIB equations of motion. The constant C in (5.60) depends on parameter $r_\star$ in (4.45) and the bare string coupling e^{ϕ_0}, $C = r_\star^2 e^{-2\phi_0}$.

In the remaining of this section we argue that our solution (5.56) have a regular horizon. It is easy to see that there will be a regular horizon at $r = r_h$, $\triangle_1(r_h) = 0$, when $F(y(r))e^{-y(r)}$ and $h(r)$ are nonzero at $r = r_h$. Really, in this case in the vicinity of r_h, we can introduce a well-defined coordinate $\eta \equiv \triangle_1$, so that the metric (5.52) can be written as

$$\begin{aligned} ds_E^2 \approx\ & c_1(r_h)^2\left[\eta^2 dt^2 - d\bar{x}^2\right] - \frac{c_1^{18}(r_h)a^2h^2(r_h)}{A^2}\ d\eta^2\\ & -c_1(r_h)^2a^2\Bigg[\frac{h(r_h)}{4}\left(d\theta_1^2+\sin^2\theta_1\ d\phi_1^2\right) \end{aligned}$$

$$+\frac{1}{4}\left(d\theta_2^2+\sin^2\theta_2\ d\phi_2^2\right)+\frac{1}{4}\left(d\psi+\sum_{i=1}^{2}\ \cos\theta_i\ d\phi_i\right)^2\Bigg], \qquad (5.61)$$

which clearly describes a nonsingular horizon at r_h, provided $c_1(r_h)$ and $h(r_h)$ are nonzero. From (5.57), as $r \to r_h$, $y \to -\infty$. So the existence of regular horizon implies

$$\begin{aligned} &F(y)\to\alpha_1 e^y\,,\\ &h(y)\to\alpha_2\,, \qquad \text{as } y\to-\infty\,, \end{aligned} \qquad (5.62)$$

for some positive constants α_i. This boundary conditions are compatible with (5.56), and allow to construct a power series solution[8]

$$\begin{aligned} F(y) &= \alpha_1 e^y\left[1+\sum_{k=1}^{\infty} q_k e^{2ky}\right]\,,\\ h(y) &= \alpha_2\left[1+\sum_{k=1}^{\infty} p_k e^{2ky}\right]\,, \qquad (-y)\gg 1\,, \end{aligned} \qquad (5.63)$$

where the first couple terms are given by

$$\begin{aligned} q_1 &= \left(\frac{\alpha_1}{A}\right)^2\left(1+\alpha_2^2\right)\,, \qquad q_2=\left(\frac{\alpha_1}{A}\right)^4\left(1+\alpha_2^2+\alpha_2^3+\alpha_2^4\right)\,,\\ p_1 &= 2\left(\frac{\alpha_1}{A}\right)^2\left(\alpha_2-1\right)\,, \qquad p_2=\left(\frac{\alpha_1}{A}\right)^4\left(\alpha_2-1\right)\left(\alpha_2^2+3\alpha_2-1\right)\,. \end{aligned} \qquad (5.64)$$

Compatibility of the boundary conditions at the regular horizon with the equations of motion is a strong hint that such regular horizon indeed exists. This should be contrasted with the non-extremal generalization of the KT geometry discussed in [8]. It is possible to show that boundary conditions at a regular horizon of the non-BPS generalization of the KT geometry with constant dilaton are actually incompatible with the equations of motion[9].

[8] The form of the series expansion is simplest to deduce by rewriting (5.56) in terms of a new variable $x \equiv \alpha_1 e^y$.

[9] On one hand, this follows indirectly from the conclusion of [9] that non-extremal deformations with constant dilaton produce *singular* horizons. One could also see this directly by repeating analysis discussed here.

Once convinced that it is consistent to impose boundary conditions of the regular horizon on (5.56), the next step is to identify restrictions on (α_1, α_2) coming from the asymptotic in the UV, where we expect to recover the extremal solution (5.60). The statement that parameters (α_1, α_2) compatible with the UV asymptotic exist at all, is highly nontrivial. We show however, that it is actually true.

From the first two equations in (5.56) and the boundary condition (5.63), it follows that both $F(y)$ and $h(y)$ are monotonic functions of y. While $F(y)$ always increases, $h(y)$ increases for $\alpha_2 > 1$ and decreases for $\alpha_2 < 1$. When $\alpha_2 = 1$, $h(y) = 1$ identically. In the UV, at the extremality, we have both h and F increasing, thus only $\alpha_2 > 1$ can be compatible with the UV asymptotic. In what follows we assume this is the case. We would like to show now that both $F(y)$ and $h(y)$ become infinitely large at finite $y \equiv y_{UV}$. This follows from the fact that as $r \to \infty$, $\triangle_1(r)$ approaches a constant — actually one, for proper normalization of the non-BPS deformation. Really, as $h(y) \gg 1$, the first equation in (5.56) is well approximated by

$$A^2 \Big[\ln\Big(F(y)h(y)\Big)\Big]'' \approx 4\,(F(y)h(y))^2\,, \tag{5.65}$$

which can be solved exactly

$$F(y)h(y) \approx AC_1\,\frac{e^{yC_1+C_2}}{1-e^{2(yC_1+C_2)}}\,, \tag{5.66}$$

where C_1 and C_2 are integration constants. C_1 is positive as $F(y)h(y)$ is always positive, thus $F(y)h(y)$ blows up at finite $y_{UV} = -C_2/C_1$. It is straightforward to check, that given (5.66), from the second equation in (5.56),

$$h(y) \approx -2\log\Big[1-e^{2(yC_1+C_2)}\Big]\,, \qquad \text{as } y \to y_{UV} - 0\,. \tag{5.67}$$

We already mentioned that proper normalization of the non-extremal deformation requires $y_{UV} \equiv \log[\triangle_1(r \to \infty)] = 0$, while we are finding that $y_{UV} = -C_2/C_1$. This is not a contradiction. Note that equations (5.56) are invariant under an arbitrary finite shift of y. Shifts in y can be absorbed

into the multiplicative normalization of α_1. Thus $y_{UV} = 0$ uniquely fixes α_1, and the only free parameter is α_2 constraint by $\alpha_2 > 1$. We would like to show now that for any $\alpha_2 > 1$ we have asymptotically (5.60) for specific C determined by α_2. This is easiest to see from equation (5.59). We are looking for the most general solutions of (5.59), such that

$$F(h) \to \infty\,, \qquad \text{as } h \to \infty\,. \tag{5.68}$$

Let's assume first that $A = 0$. With (5.68), solution of (5.59) is a power series in $e^{-h/2}$

$$F(h) = d_0 e^{h/2} h^{-1/2}\Big[1 + d_1 e^{-h/2} h^{3/2}\Big(1 + O\left(1/h\right)\Big) + O\Big(e^{-h}\Big)\Big]\,, \tag{5.69}$$

where d_0 and d_1 are arbitrary integration constants. It is easy to see that (5.69) also solves (5.68) with $A \neq 0$ to the specified order, and the corrections to (5.59) from finite A, show up as $\delta F(h) \sim A^2 e^{-h/2}$ corrections to (5.69). Since the latter corrections are subdominant, they would generically fix d_1. This is indeed what we find

$$F(h) = d_0 e^{h/2} h^{-1/2}\Big[1 - \frac{A^2}{4d_0^2} e^{-h} h^{-2}\Big(1 + O\left(1/h\right)\Big) + O\Big(e^{-2h}\Big)\Big]\,. \tag{5.70}$$

That is, d_1 in (5.69) is fixed to be zero for $A \neq 0$. Eq. (5.70) reproduces the extremal solution in the UV if we identify C of (5.60) with d_0. Clearly d_0 depends on (α_2, A). The A dependence is trivial. Note that the A dependence of (5.56) drops out if we redefine $F(y) \to F(y)A$. Thus

$$C \equiv d_0 = A\ \mathcal{F}(\alpha_2)\,, \tag{5.71}$$

where $\mathcal{F}(\alpha_2)$ is some specific function. Unfortunately, we do not know the relevant exact analytical solution of (5.56), and thus we can not determine this function explicitly[10]. Recall that for the extremal solution $C = r_*^2 e^{-2\phi_0}$, so from (5.71) we identify

$$\alpha_2 = \mathcal{F}^{-1}\left(\frac{r_*^2}{A} e^{-2\phi_0}\right)\,, \tag{5.72}$$

[10]Unlike solutions discussed in [10] there is no small parameter here which could be used to set up a perturbation theory.

where we used $\mathcal{F}^{-1}$ to denote an inverse function to $\mathcal{F}$. We argued previously that to have a regular horizon with proper UV asymptotic $\alpha_2 > 1$, thus

$$\mathcal{F}^{-1}\left(\frac{r_*^2}{A}e^{-2\phi_0}\right) > 1\,, \tag{5.73}$$

indirectly determines relation between the non-extremality parameter A and the scale of the chiral symmetry breaking of the extremal solution $\sim r_*$.

Additionally, numerical results support our belief that solutions exist which properly interpolate between the boundary conditions (5.63), (5.64) at the horizon and the appropriate UV asymptotic (5.60). To do so, we set initial conditions for F and h using the p_1, q_1 terms in the series (5.63) at a value of $x = e^y$ such that the correction was small. Then we solved the first two equations of (5.56) numerically (using the variable x). For all the cases studied, both $F \to \infty$ and $h \to \infty$ at a finite value of x, which can be shifted to $x = 1$ by the multiplicative normalization of α_1 discussed above. It is then possible to check the UV asymptotic by making a log-log plot of $Fe^{-h/2}$ versus h near $x = 1$ and find the power law slope. Again, in all cases studied, we found a power law between $h^{-0.49}$ and $h^{-1/2}$, correctly reproducing the UV asymptotic (5.60).

To summarize, in this section we constructed a family of non-extremal deformations of the singular CV-MN solution. We argued that our deformations have a regular Schwarzschild horizon, and showed that given the scale of a naked singularity of the chirally symmetric CV-MN solution, r_*, its non-singular non-extremal deformation exists only for a range of non-extremality parameter A determined from (5.73).

6 Discussion

In this paper we considered supersymmetric compactifications of type IIB string theory on resolved conifolds with torsion. We extended the no-go theorem stated previously for compact Hermitian six dimensional manifolds in [17] to the non-compact manifolds with the complex structure of the resolved

conifold, and showed that nonvanishing torsion leads to the naked singularities in the geometry. In classifying supersymmetric compactifications we have not found the nonsingular Chamseddine-Volkov-Maldacena-Nunez solution [11, 12, 5], which was argued in [23] to be in the class of string backgrounds on Hermitian manifolds with torsion. As the infrared (small r) geometry of [11, 12, 5] is that of the deformed conifold, and thus its complex structure is different from the complex structure of the resolved conifold, the latter fact is not a contradiction. Classification of supersymmetric compactifications on deformed conifolds with torsion should recover solution of [11, 12, 5] and might uncover other interesting nonsingular backgrounds. We plan to return to this problem in the future.

We discussed non-extremal generalizations of the CV-MN solution describing the unbroken symmetry phase of the dual Yang Mills theory. We argued (though not proven rigorously) that the simplest solution we found have regular horizon which develops only above some critical non-extremality. This is in accord with the gauge theory where the chiral symmetry restoration occurs at finite temperature. There are lots of open questions. First of all, it would be extremely interesting to determine the exact analytical form of the function $\mathcal{F}$ in (5.71), and thus *explicitly* prove the restriction on the non-extremality coming from the boundary conditions of the regular horizon. Maybe it is possible to solve (5.56) exactly[11] ? We did not discuss the thermodynamical quantities of the background. It would also be interesting to study the critical black hole in this non-extremal geometry, corresponding to the gauge theory at the phase transition. Finally, the non-extremal generalization we found is rather special. It is thus interesting to find out how generic it is, and if other solutions exist, what is their interpretation.

[11] It is possible to reduce the first two second order equations in (5.56) to first order equations; the third equation becomes then algebraic.

Acknowledgments

We wish to thank Jerome Gauntlett, Steven Gubser, Sunny Itzhaki, Igor Klebanov, Andrei Mikhailov, Amanda Peet and Joe Polchinski for valuable discussions. The work of AB was supported in part by NSF grants PHY97-22022 and PHY99-07949. The work of AF was supported by a National Science Foundation Graduate Research Fellowship.

Appendix

We follow notations of [15]. The metric signature is mostly minus and the Clifford algebra is

$$\{\hat{\gamma}^{r_i}, \hat{\gamma}^{r_j}\} = 2\eta^{r_i r_j} \,, \tag{6.74}$$

where r_i is the tangent space index and $\eta^{r_i r_j}$ is the Minkowski metric. We define

$$\begin{aligned} &\hat{\gamma}^{r_1 \cdots r_k} = \hat{\gamma}^{[r1}\hat{\gamma}^{r_2} \cdots \hat{\gamma}^{r_k]} \,, \\ &\hat{\gamma}^{11} = \hat{\gamma}^1 \cdots \hat{\gamma}^{10} \,, \end{aligned} \tag{6.75}$$

where symmetrization of indices is carried out with weight one: $[ab] = \frac{1}{2!}(ab-ba)$. The notation $\hat{\gamma}$ indicates a ten-dimensional quantity, while g indicates either four or six dimensional quantity. In a Majorana representation $\hat{\gamma}^1$ is antisymmetric and imaginary, and $\hat{\gamma}^2$ to $\hat{\gamma}^{10}$ are symmetric and imaginary. When r indices are used, $\hat{\gamma}$ matrices are purely numerical (independent of the coordinates). When they are converted to greek indices with the 10-bein $\hat{e}^r_M$ or its inverse $\hat{e}^M_r$, they become field dependent functions. We use roman subindices $M, N, ... = 1, ..., 10$; $\mu, \nu = 1, 2, 3, 4$ and $m, n, ... = 5, ..., 10$.

We consider warped product geometries of the form

$$d\hat{s}^2_{10} = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu - e^{2B(y)}g_{mn}dy^m dy^n \,, \tag{6.76}$$

where $A(y)$, are $B(y)$ are warp factors that depend only on 6D indices. The minus sign in (6.76) allows us to have six-dimensional metric of positive

signature. The natural relation between 10- and 4- (6-) beins is

$$\hat{e}^r_\mu = e^A e^r_\mu\,, \qquad \hat{e}^r_m = i e^B e^r_m\,, \tag{6.77}$$

where i in (6.77) accounts for the change of signature $\hat{g}_{mn} = -e^{2B} g_{mn}$. From (6.77),

$$\hat{\gamma}_m = i e^B \gamma_m\,, \qquad \hat{\gamma}^m = -i e^{-B} \gamma^m\,, \tag{6.78}$$

with

$$\{\gamma_m, \gamma_n\} = 2 g_{mn}\,. \tag{6.79}$$

Note that the γ_m matrices are symmetric and real.

For the k-forms we use notation $F_{(k)} = \frac{1}{k!} F_{M_1 \cdots M_k} dx^{M_1} \wedge \cdots \wedge dx^{M_k}$. We define

$$\hat{\not{F}}_{(k)} = \frac{1}{k!} F_{M_1 \cdots M_k} \hat{\gamma}^{M_1 \cdots M_k}\,. \tag{6.80}$$

Note that if the k-form is nonzero only in six dimensions,

$$\hat{\not{F}}_{(k)} = (-i)^k e^{-kB} \not{F}_{(k)}\,. \tag{6.81}$$

Hodge duals are defined as

$$(\hat{\star} F)_{M_{k+1} \ldots M_{10}} = \frac{\sqrt{|\hat{g}|}}{k!} \hat{\epsilon}_{M_{k+1} \ldots M_{10}}{}^{M_1 \cdots M_k} F_{M_1 \cdots M_k}\,, \tag{6.82}$$

and similarly for the six-dimensional Hodge dual $\star$. We take convention $\hat{\epsilon}_{12 \cdots 10} = +1$; also $\epsilon_{5 \cdots 10} = +1$. We also need the adjoint exterior derivative operator, which in six-dimensions is defined as

$$d^+ \equiv \star d \star\,. \tag{6.83}$$

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